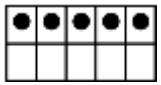
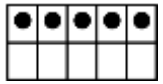
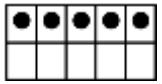
LAKESIDE
PRIMARY ACADEMY

Calculation Policy

Please note that the concrete examples given below are not an exhaustive list. Other equipment may be used and should be used in order for children to deepen their understanding.

Progression in the use of concrete apparatus						
Foundation	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Real-life objects	Real-life objects	Real-life objects	Real-life objects	Real-life objects	Real-life objects	Real-life objects
0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards
Number track to 10	Number line to 20	Number line to 100	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
Numbered counting stick	Counting stick	Counting stick	Counting stick	Counting stick	Counting stick	Counting stick
Tens frame 	Tens frame 	Tens frame 				
Place value charts – Tens and ones	Place value charts – Tens and ones	Place value charts – Hundreds, tens and ones	Place value charts – Thousands, hundreds, tens and ones	Place value charts – Ten thousands, thousands, hundreds, tens, ones and tenths	Place value charts to a million and three decimal places	Place value charts to 10 million and three decimal places
Interlocking cubes - Use one colour to represent one amount	Interlocking cubes - Use one colour to represent one amount	Dienes	Dienes	Dienes	Dienes	Dienes
			Place value counters	Place value counters	Place value counters	Place value counters
	Place value arrow cards – tens and ones	Place value arrow cards – tens and ones	Place value arrow cards – H, T, O	Place value arrow cards – Th, H, T, O	Place value arrow cards	Place value arrow cards

Foundation	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Part-part-whole mat	Part-part-whole mat	Part-part-whole mat	Part-part-whole model	Part-part-whole model	Part-part-whole model	Part-part-whole model
Bar model with real-life objects	Bar model with real life objects/pictorial objects/representative objects eg. counters	Bar model with counters /Dienes progressing to numbers	Bar model with numbers	Bar model with numbers	Bar model with numbers	Bar model with numbers
Please also refer to our Progression in Bar Modelling guidance for further detail and examples.						
Bead strings – ten	Bead strings - twenty	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred
Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes
			Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods
Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters
Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount

Please see the National Curriculum for suggested year group objectives
(https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf)

Maths Working Walls		
All working walls will display the following:		
Concrete	Use a real-life representation of the concept which children can see, touch and feel.	
Pictorial	Show a pictorial representation of the concept.	
Abstract	Show the mathematical representation of the concept.	6 x 2 = 12 2 x 6 = 12 12 ÷ 2 = 6 12 ÷ 6 = 2
Vocabulary	Use vocabulary related to the concept	Multiply, times, repeated addition, array, divide, group, multiples, factors.
The following may also be seen:		
Practise	Encourage children to practice the concept. ***Interactive opportunity*** – ask children to respond to questions, encourage them to add what they know, leave homework for children to take to master the	1 x 2 = 2 2 x 2 = 4 3 x 2 = 6 etc.
Challenge	Set a challenge to be solved. ***Interactive opportunity*** – leave real-life objects or manipulatives for children to use to help solve the	How many different ways can 12 eggs be arranged into arrays?

Progression in the teaching of counting in the Foundation Stage

Pre-counting	**Ordering**	**One to one correspondence**	**Cardinality (Knowing the final number counted is the total number of objects)**
The key focus in pre-counting is an understanding of the concepts more, less and the same and an appreciation of how these are related. Children at this stage develop these concepts by comparison and no counting is involved.	Count by reciting the number names in order forwards and backwards from any starting point.	One number word has to be matched to each and every object. Lack of coordination is a source of potential error – it helps if children move the objects as they count, use large rhythmic movements, or clap as they count.	Count out a number of objects from a larger collection. Know the number they stop counting at will give the total number of objects.
Pre-counting ideas *Provide children with opportunities to sort groups of objects explicitly using the language of **more** and **less.***  *Which group of apples has the most?* *Which group of apples has the least?*	**Ordering ideas** *Provide children with opportunities to count orally on a daily basis. Rote count so that children are able to understand number order and can hear the rhythm and pattern. Use a drum or clap to keep the beat.* 	**One to one correspondence ideas** *Play counting games together moving along a track, play games involving amounts such as knocking down skittles.* *Use traditional counting songs throughout the day ensuring children have the visual/kinaesthetic resources eg. 5 little ducks, 10 green bottles*	**Cardinal counting ideas**  *How many bananas are in my fruit bowl? Allow children to physically handle the fruit.* *Provide children with objects to point to and move as they count and say the numbers.*

Progression in the teaching of counting in the Foundation Stage

Subitising (recognise small numbers without counting them)

Children need to recognise small amounts without counting them eg. dot patterns

on dice, dots on tens frames, dominoes and playing cards as well as small groups of randomly arranged shapes stuck on cards.

Abstraction

You can count anything – visible objects, hidden objects, imaginary objects, sounds etc. Children find it harder to count

things they cannot move (because the objects are fixed), touch (they are at a distance), see, that move around. Children also find it difficult to count a

mix of different objects, or similar objects of very different sizes.

Conservation of number – MASTERY!

Ultimately children need to realise that when objects are rearranged the number of them stays the same.

End of year counting expectations

count reliably to 20

count reliably up to 10 everyday objects

estimate a number of objects then check by counting

use ordinal numbers in context eg first, second, third

count in twos, fives and tens

Order numbers 1-20

Say 1 more/ 1 less than a given number to 20

Subitising ideas

Provide children with opportunities to count by recognising amounts.

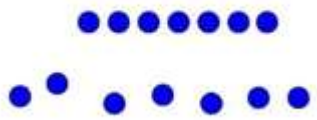
Abstraction ideas

How many pigs are in this picture?

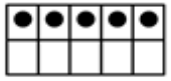
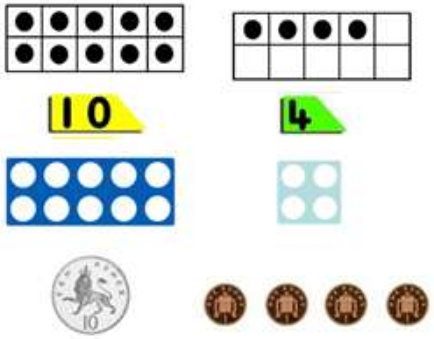
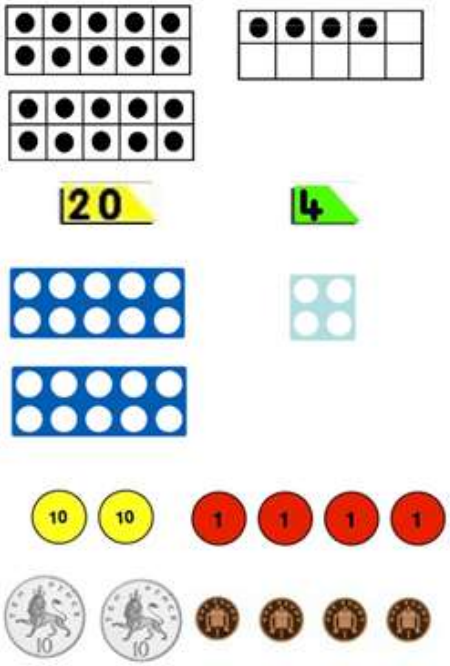
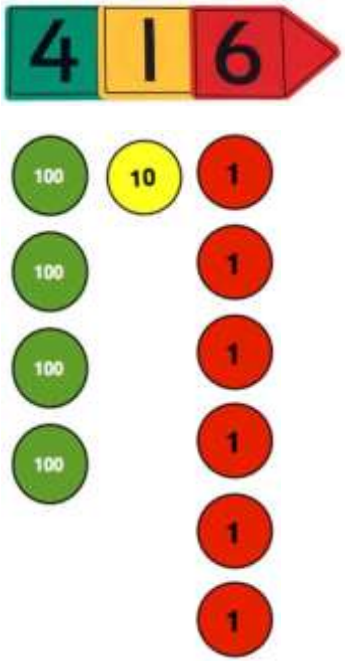
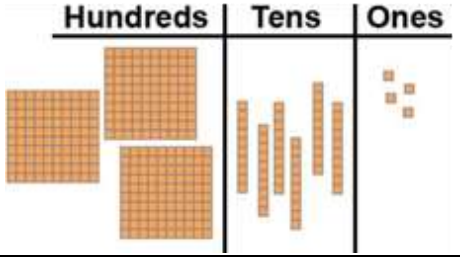
Provide children with a variety of objects to count.

Conservation of number

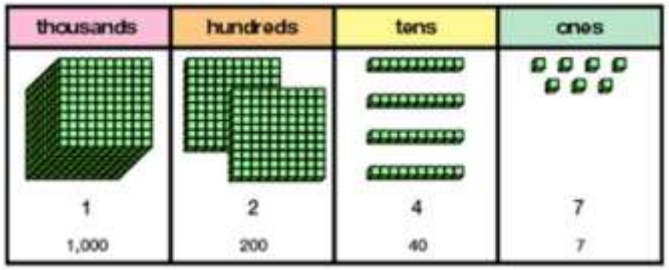
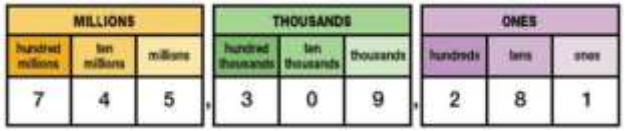
The amount is 'seven' and does not change.



Progression in the teaching of place value

Understanding ten	Understanding numbers up to 20	Understanding numbers up to one hundred	Understanding numbers up to one thousand
A TENS FRAME is a simple maths tool that helps children: Keep track of counting See number relationships Learn addition to 10 Understand place value Use ***tens frames*** flash cards daily to ensure children recognise amounts. Use empty ***tens frames*** to fill with counters to enable children to understand number relationships. Either fill the ***tens frame*** in pairs or in rows. In rows shows 5 as a benchmark. Children can easily see more than 5 or less.  Setting the counters in pairs, naturally allows the children to see addition concepts. Include other visual images such as dice, cards, dominoes etc.   	'Ten' is the building block of our Base 10 numeration system. Young children can usually 'read' two-digit numbers long before they understand the effect the placement of each digit has on its numerical value. A child might be able to correctly read 62 as sixty-two and 26 as twenty-six, and even know which number is larger, without understanding why the numbers are of differing values. Ten-frames can provide a first step into understanding two-digit numbers simply by the introduction of a second frame. Placing the second frame to the right of the first frame, and later introducing numeral cards, will further assist the development of place-value understanding. 	Continue developing place value through the use of ***tens frames***.   Use bundles of straws to show value of tens and ones.	Continue developing place value through the use of manipulatives.  Use Dienes blocks and place value charts 

Progression in the teaching of place value (continued)

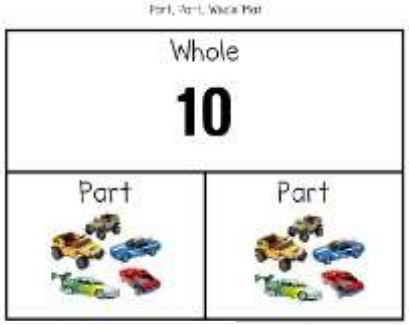
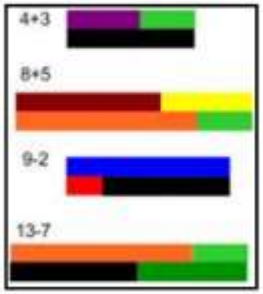
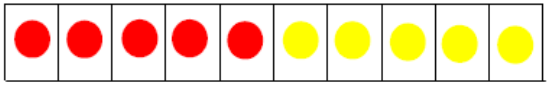
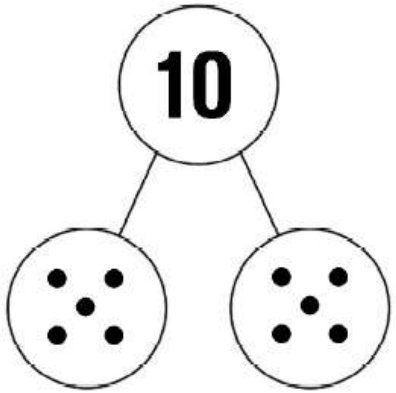
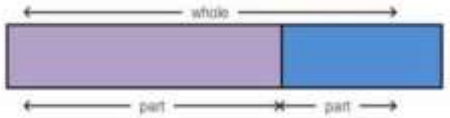
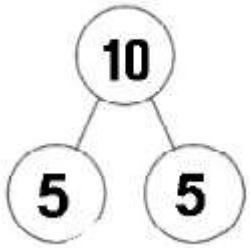
Understanding numbers up to ten thousand	Understanding numbers up to one million including decimals	Understanding numbers beyond one million including decimals
Continue developing place value through the use of manipulatives. Place value arrow cards Place value counters Dienes blocks Place value charts  	Continue developing place value through the use of manipulatives. Place value arrow cards Place value counters (including decimal counters) Dienes blocks Place value charts  	Continue developing place value through the use of manipulatives. Place value arrow cards Place value counters (including decimals counters) Dienes blocks Place value charts 

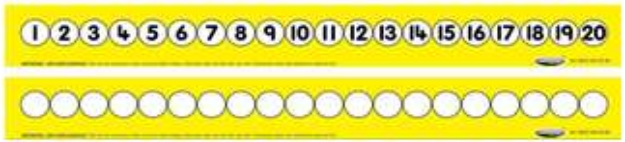
Progression in the teaching of calculations

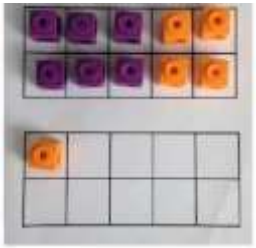
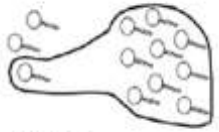
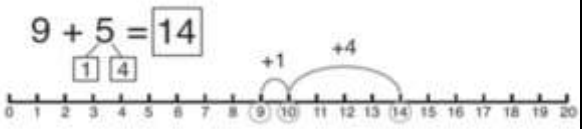
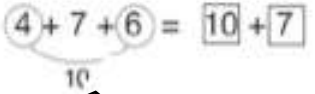
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

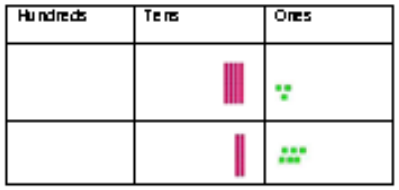
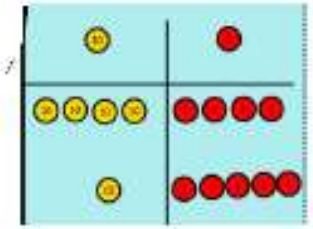
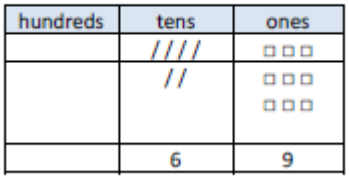
Progression in the teaching of calculations

Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Combine two parts to make a whole model. Part-part-whole model Teach the children that the cubes/counters represent the real-life objects.	  Once are familiar with Cuisenaire, these can be used to represent addition and subtraction linked to part-part-whole.  	   Part + Part = Whole Whole – Part = Part	 Use the part-part whole diagram as shown above to move into the abstract. 5 + 5 = 10 10 = 5 + 5 Use the term ‘number sentence’.

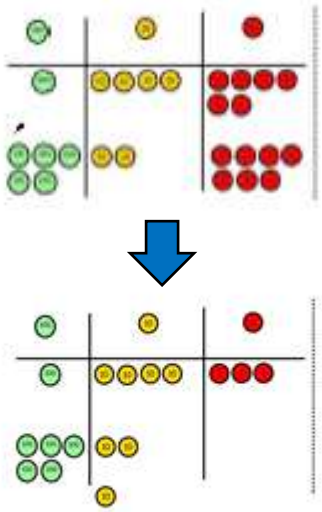
Use cubes to add two numbers together as a group or in a bar.			
Start at the larger number and count on Start with the larger number on the bead string then count on 1 by 1 to find the total	 Use counters on a number track to count on. 	 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	4+7=11 Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10. Start with the bigger number and use the smaller number to make 10.	 6 + 5 = 11 	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits. Encourage children to use known facts.	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	Add together three groups of objects. Draw a picture to recombine the groups to make 10.  4 + 6 + 7 = 17	 Combine the two numbers that make 10 and then add on the remainder.

Column method – no regrouping Use Dienes to add tens and ones before moving on to place value counters.	 43 + 26 _____ 	After practically using the base 10 blocks and place value counters, children can draw the Dienes to help them to solve addition calculations.  	21 +42 63 Only select numbers which do not involve regrouping.

Column method – regrouping

Make both numbers on a place value grid.



146
+527

Add up the ones, regrouping for a ten when there are ten ones etc.

This can also be done with Dienes to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

Continue using place value counters as children begin to work with decimals.

Children can then move on to drawing a pictorial representation of the columns and place value counters to further support their learning and understanding.

hundreds	tens	ones
/	////	□□□□□ □
/////	//	□□□□□ □□
6	6	3
	1	

$$\begin{array}{r} 536 \\ +\,85 \\ \hline 621 \\ \hline 11 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places.

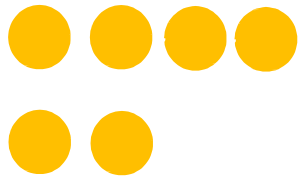
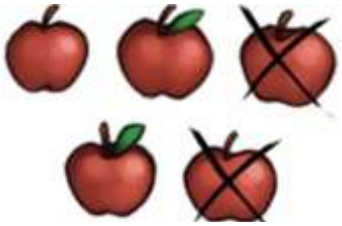
$$\begin{array}{r} 72.8 \\ +\,54.6 \\ \hline 127.4 \\ \hline 1\ 1 \end{array}$$

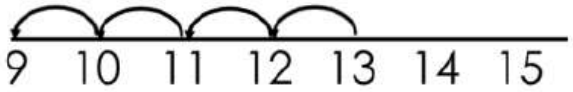
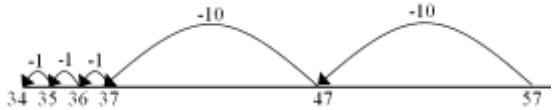
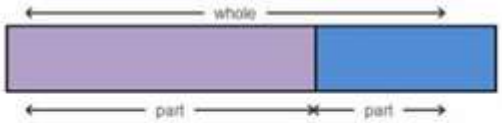
Then move onto decimals with a different number of decimal places.

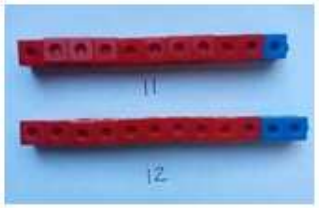
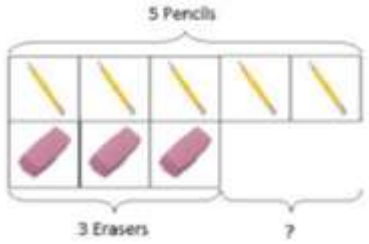
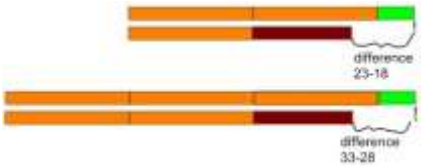
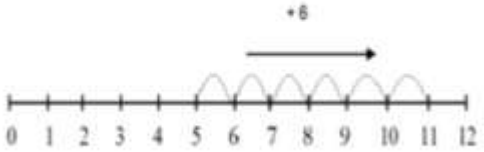
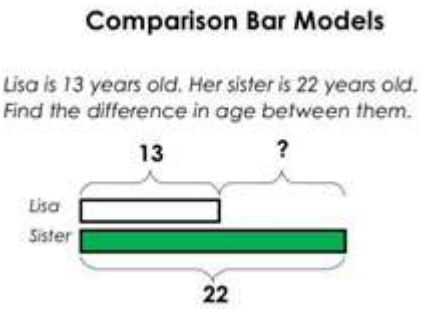
$$\begin{array}{rrcrrr} 2 & 3 & . & 3 & 6 & 1 \\ & 9 & . & 0 & 8 & \mathbf{0} \\ 5 & 9 & . & 7 & 7 & \mathbf{0} \\ + & 1 & . & 3 & \mathbf{0} & \mathbf{0} \\ \hline 9 & 3 & . & 5 & 1 & 1 \\ \hline 2 & 1 & & 2 & & \end{array}$$

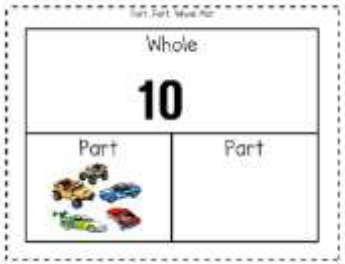
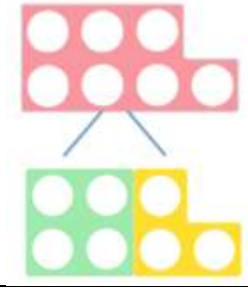
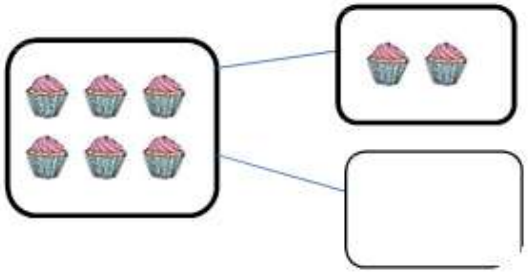
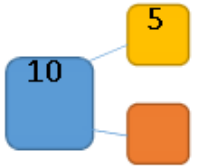
Progression in the teaching of calculations

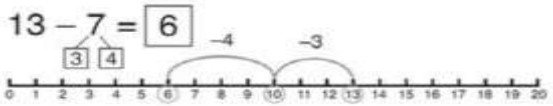
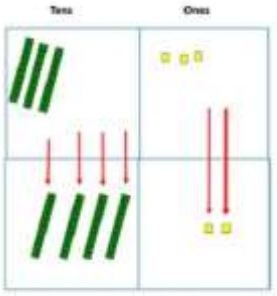
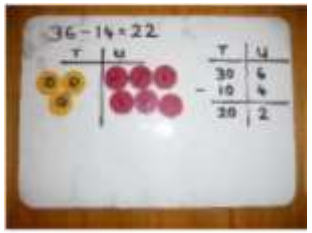
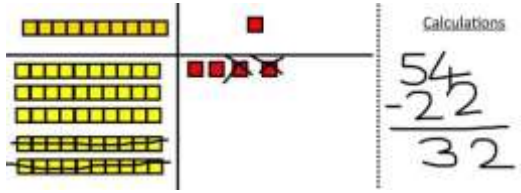
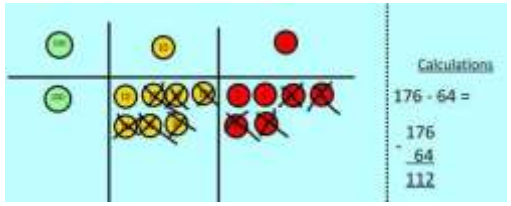
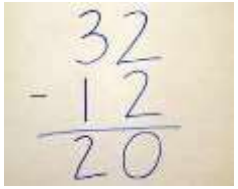
Subtraction

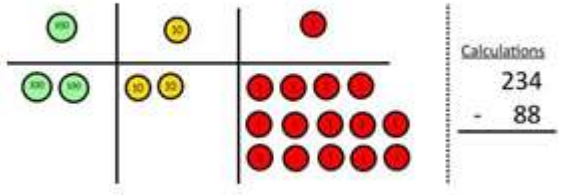
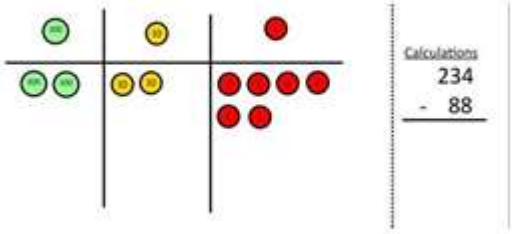
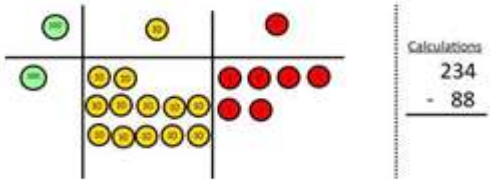
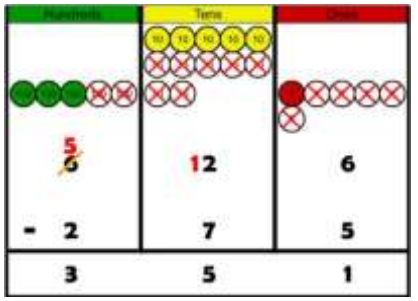
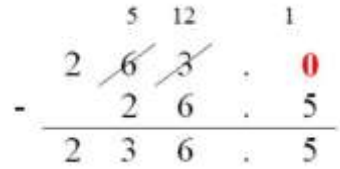
Objective and Strategies	Concrete	Pictorial	Abstract
Take away ones	Use real-life physical objects, counters, cubes etc. to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  5 – 2 = 3	4 = 6 – 2 18 - 3= 15 8 – 2 = 6

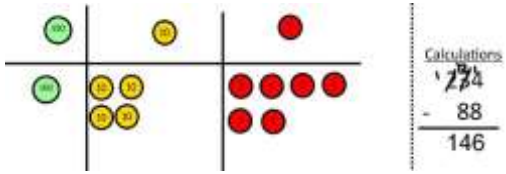
Counting back	Make the *minuend* (the number you're subtracting from)  13 – 4 Move the beads along the bead string whilst counting backwards in ones.  Use counters and move them away from the group whilst counting backwards.  Use cubes and subtract a number from the bar.	Count back on a number line or number track  Start at the minuend and count back the *subtrahend*(the number that you are subtracting) showing the jumps on the number line.  Use a bar model:  Part + Part = Whole Whole – Part = Part	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. Children will need regular practice counting backwards.

Find the difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference:  Use basic bar models with objects to find the difference:  Use Numicon:  Use Cuisenaire rods:  	Count on using a number line from the smallest to the greatest number.  Draw a bar model:  	Hannah has 23 pencils. Helen has 15 pencils. Find the difference in the number of pencils.

Part Part Whole	Link to addition- use the part part whole method to help explain the inverse. If 10 is the whole and 5 is one of the parts. What is the other part? 10 - 5 = or 10 - ? = 5  Part - Whole Use Numicon: 	Use pictures of objects to represent the part part whole method: 	Move to using numbers with the part part whole method:  10 - 5 = 5 or 5 = 10 - ?

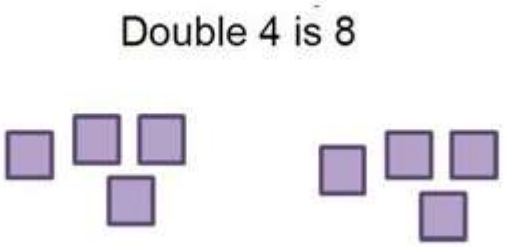
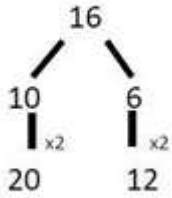
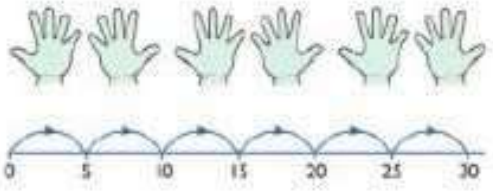
Make ten	14 – 5 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8 = How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping	75- 42 =  Use Dienes to make the minuend (usually the bigger number) then take the subtrahend (usually the smaller number) away. Show how you partition numbers to subtract. Again make the larger number first.  	Draw the Dienes or place value counters alongside the written calculation to help to show working.   	This will lead to a clear written column subtraction. 

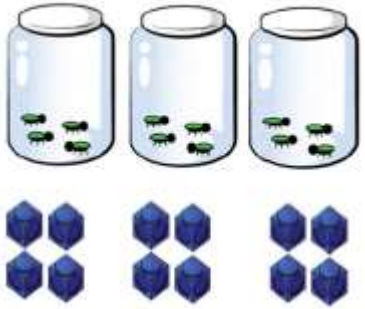
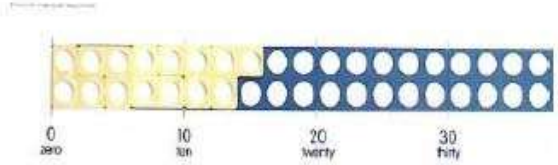
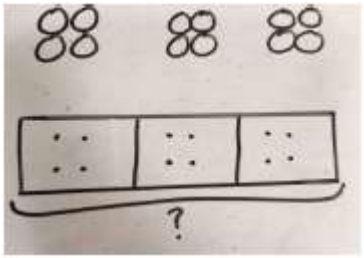
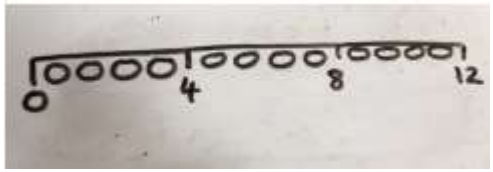
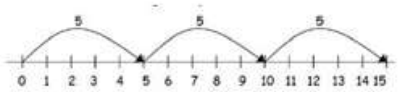
Column method with regrouping	Make the larger number with the Dienes or place value counters (only use place value counters when understanding of place value is secure). Start with the ones, can I take away 8 ones from 4 ones easily? I need to exchange one of my tens for ten ones. *Make the value of each digit explicit 'e.g. 3 tens subtract 4 tens' not just '3 subtract 4'.*  Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  	Draw the counters onto a place value grid and show what has been taken away by crossing the counters out as well as clearly showing the exchanges made.  When confident, children can find their own way to record the exchange/regrouping.	 This will lead to an understanding of subtracting any number including decimals. 

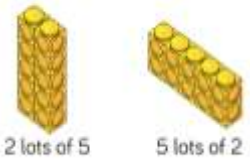
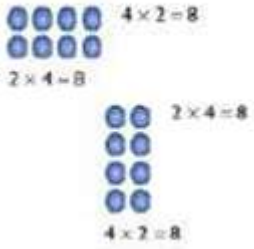
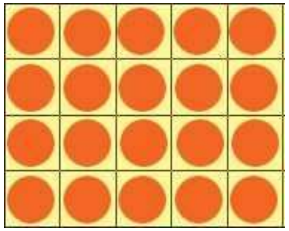
	Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Sometimes it might be better to leave showing them this until they have grasped the regrouping. Cross out the numbers when exchanging and show where we write our new amount.		

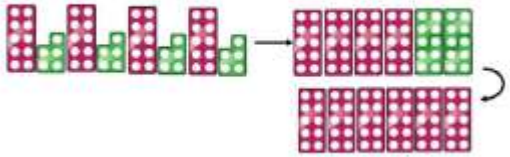
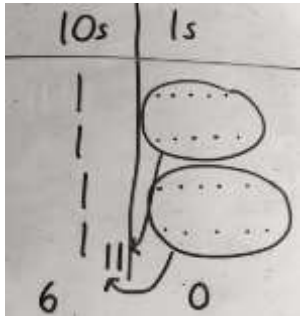
Progression in the teaching of calculations

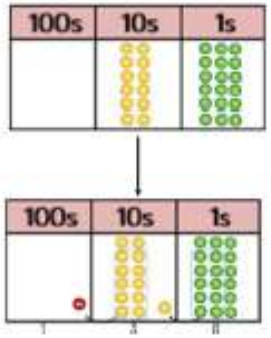
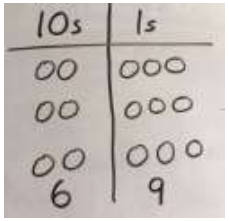
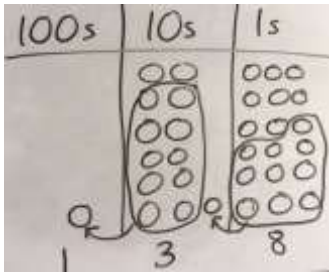
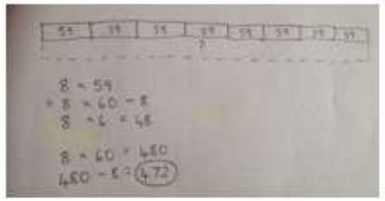
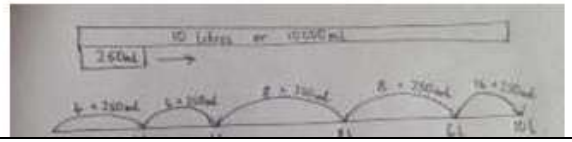
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  **5x2=10**	Draw pictures to show how to double a number. 	Doubling by using known number facts and partitioning:  Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count aloud in multiples of a number. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

Repeated addition	Use different objects to add equal groups: 4+4+4 3x4 There are 3 equal groups, with 4 in each group.   Use number tracks with Cuisenaire and Numicon to show repeated groups:  3 + 3 + 3 + 3 + 3 5 x 3 = 15  0 zero 10 ten 20 twenty 30 thirty	Represent the concrete apparatus in a picture and use bar models:    5 + 5 + 5 = 15	3x4=12 4+4+4=12 5x3=15

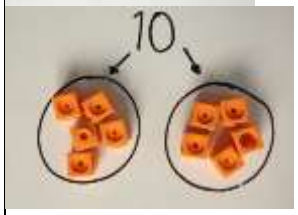
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.   Use arrays to illustrate commutativity:  	Rotate arrays to find commutative multiplication sentences:  Link arrays to the area of a rectangle. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

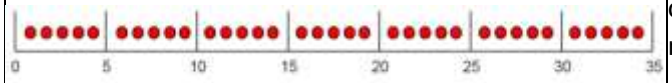
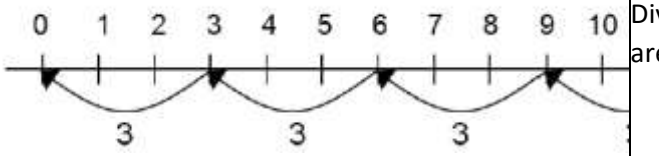
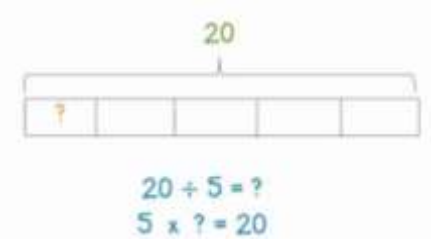
Partition to multiply (mental method)	Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60

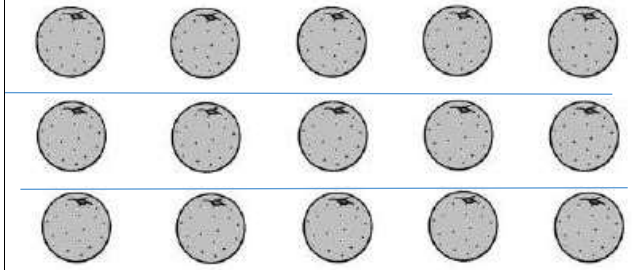
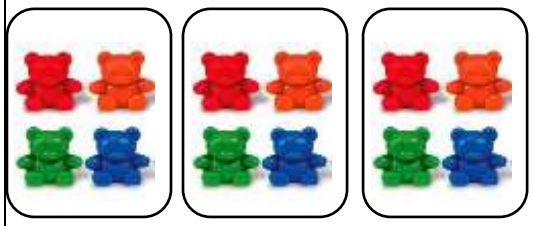
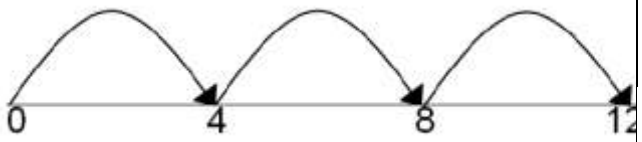
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  23x3 It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.  23x6	Children to represent the counters pictorially. 23x3   23x6 Bar modelling and number lines should be used to support when solving problems with multiplication alongside the formal written methods.  	Following Inspire manuals for the stage each class is working at, children will represent written multiplication like this:  When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract: To get 744 children have solved 6 × 124. To get 2480 they have solved 20 × 124.

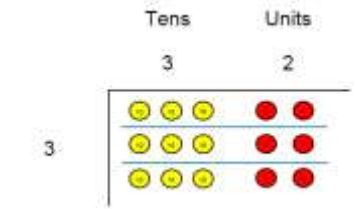
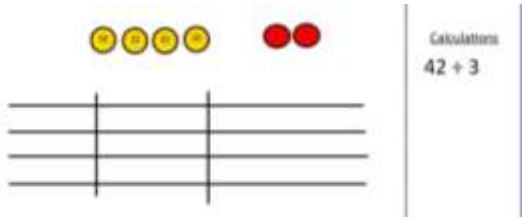
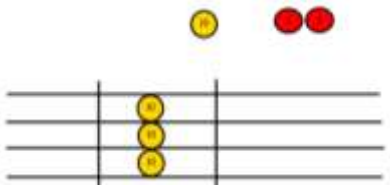
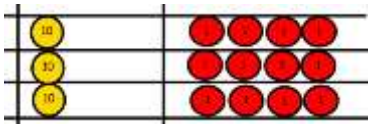
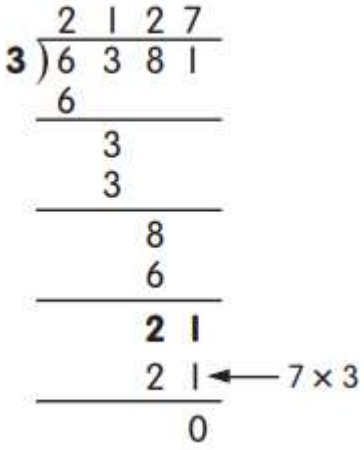
Progression in the teaching of calculations

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Division as sharing	  I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3

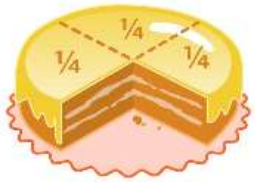
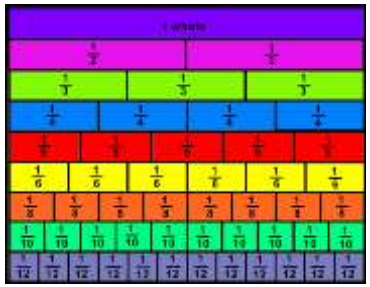
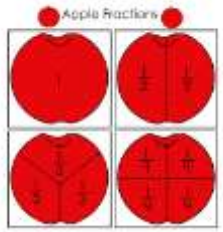
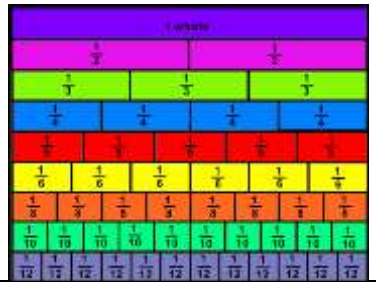
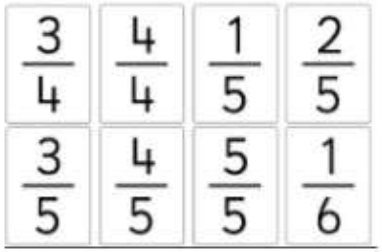
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  10 ÷ 2 '10 divided into groups of 2' 35 ÷ 5 '35 divided into groups of 5'   Cuisenaire rods on the number tracks can be used in a similar way – 'how many 5s does it take to get to 35?'	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

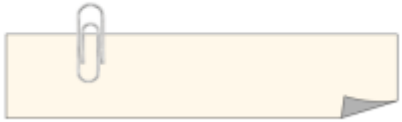
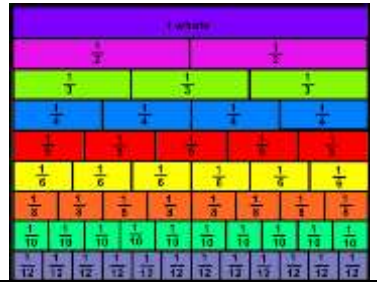
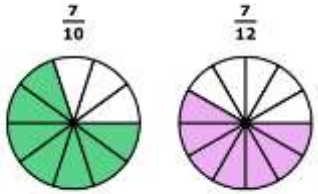
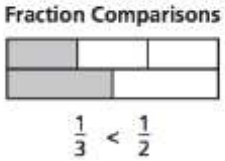
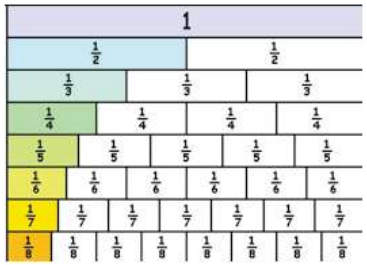
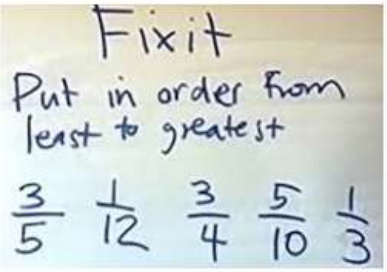
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a  remainder. Draw dots and group them to divide an amount and clearly show a remainder.  	Complete written divisions and show the remainder using r.   8 ones ÷ 2 = 4 ones with no remainder Quotient = 4 ones Remainder = 0 ones $2\overline{)8}$ 4, 8, 0

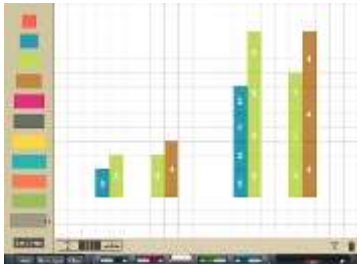
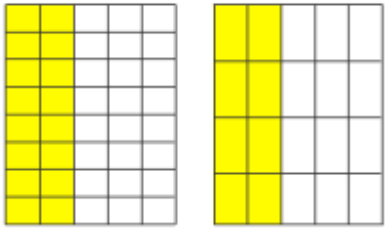
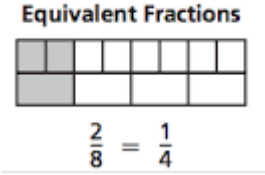
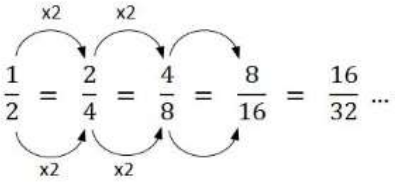
Written method of division Get children used to the practical method before introducing the written method. **Really helpful video of long division here:** http://www.mathsnoproblem.co.uk/parent-videos	 Use place value counters or Dienes to divide using the bus stop method  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.		

Progression in the teaching of calculations

Fractions

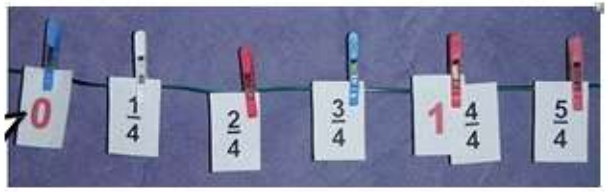
Objective and Strategies	Concrete	Pictorial	Abstract
Recognising fractions Children should be shown fractions in a variety of different ways and shapes, e.g. shapes , paper cups, paper plates, paper fraction discs (from staff meeting) We want children to see the relationship between written fractions and part-whole. Use a fraction wall with increasing variety of unit fractions to recognise and name fractions. A fraction wall cut into separate strips will allow pupils to lay them side by side and compare fifths with eighths for example.	     	  Fractions represented on number lines after practical work with fraction strips:  Pictures of fraction walls: 	 **'one fifth'** **'two thirds'**

	Use Cuisenaire rods: Watch these videos to see children using Cuisenaire for fractions. https://www.ncetm.org.uk/resources/28929 **In the first one, they've chosen one of the rods to be '1' and then they're working out the value of smaller rods based on the relative size.**		
Comparing and ordering fractions	Use a strip of A4 paper and a paper clip to practice comparing and ordering fractions along an imaginary number line.  Ask questions like "show me a half"; "show me a quarter", "show me a third", "show me two-thirds". Use the strip to represent different parameters: 0 to 1; 0 to 2; 1 to 2 etc. Use the strip to represent 1L, £1, £2, 1m etc to represent and compare fractions of continuous quantities. Use a fraction wall to compare unit and non-unit fractions. Use the < > to compare fractions. Ask pupils to generalise about how to estimate if a fraction is greater than or less than a half, quarter, three quarters and other fractions. 	Compare shaded parts of shapes:  Bar models:  Fraction wall: 	Compare $\frac{3}{4}$ & $\frac{2}{3}$ The LCM of 3 and 4 is 12 $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ $\frac{9}{12} > \frac{8}{12}$ 

	Use Cuisenaire or coloured rods to compare different coloured rods in relation to a given rod. Watch the videos above of Class 3 (Y4 and Y5) comparing fractions using **Cuisenaire**.		
Equivalence	Pupils should experience and compare fair sharing situations: Two chocolate bars shared between four children will provide the same number of bars for each person as one chocolate bar shared between two children. Pupils can explore how many chocolate bars are needed for four children or six children etc. Or, one pizza shared equally between three children will give the same number of slices as two pizzas shared between six children. Or, a bag of 40 sweets shared equally between four children will lead to the same number per child as 20 sweets shared equally between two children. (Or just counters!) Fraction wall (again) Cuisenaire rods:  Making a fraction and scaling up to find equivalent one.	Provide pupils with opportunities to observe shapes shaded as equivalent fractions. Encourage pupils to compare by asking "What do you notice?" or "What's the same, what's different about these shapes?" e.g.  Bar models: 	Using multiplication and division: 

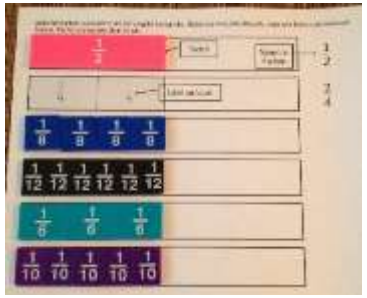
Adding fractions

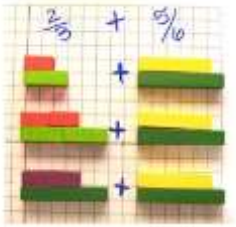
Count in fraction steps using real objects and a number line.



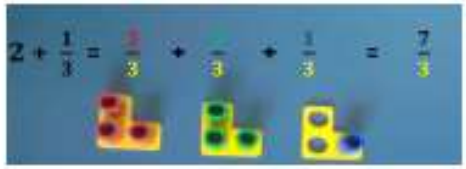
Use concrete objects such as:

a fraction wall with moveable pieces, fraction cards (NCETM) or Cuisenaire rods

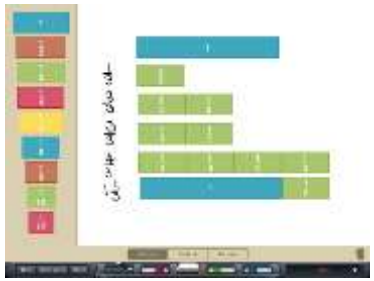




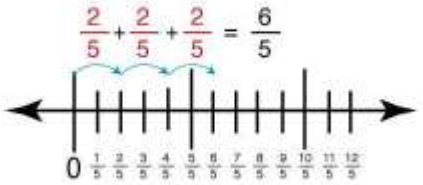
Numicon



‘Rods’ app



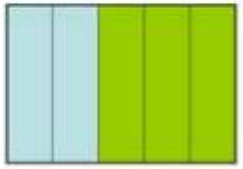
Use a number line to represent adding fractions pictorially:



Use diagrams of shapes:

Addition of fractions with the same denominator

$\frac{2}{5} + \frac{3}{5} = \frac{5}{5}$

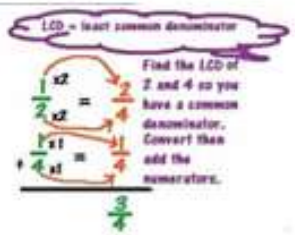


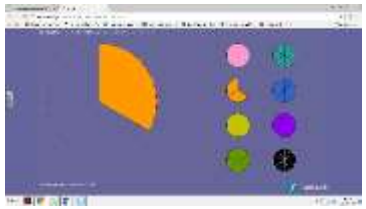
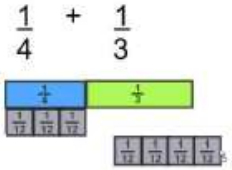
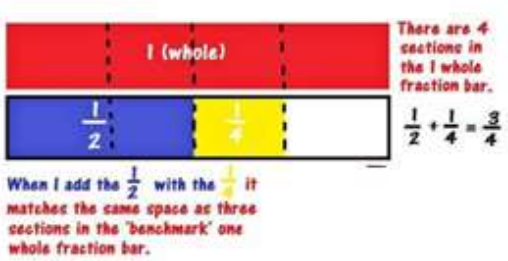
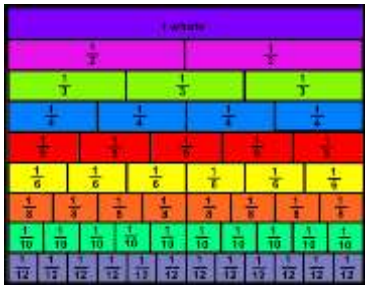
$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$

$\frac{1}{4} + \frac{1}{3} =$

$\frac{1 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4}$

$\frac{3}{12} + \frac{4}{12}$



Fraction circles. For ITP use www.taw.org.uk)		Use bar models:   Use fraction walls: 	

<table>
<tr><td>Subtracting fractions</td><td colspan="3">Use the same strategies as when adding fractions, ensuring that it is clear that you are starting with the minuend and removing/crossing out the subtrahend.</td></tr>
<tr><td>Multiplication of fractions

Note – sometimes easier to think of ‘x’ as ‘of’

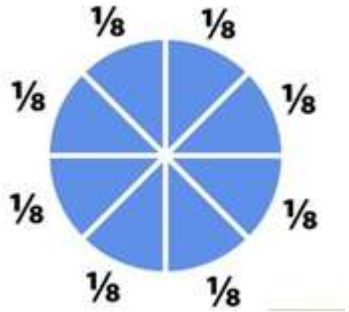
e.g. 1/2 x5 = 1/2 of 5

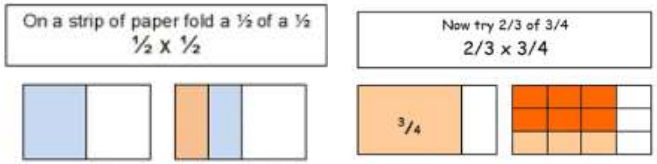
Use different objects and shapes so that children don’t just relate it to one particular shape.</td><td>Count in fraction steps (repeated addition)

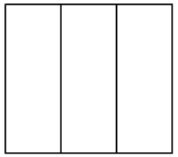
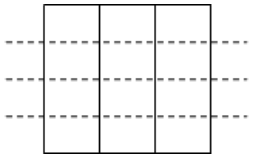
 

Use fractions of whole objects and shapes.

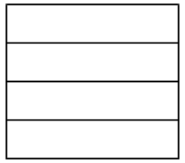
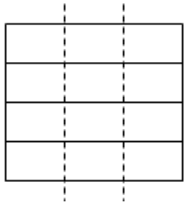
Ask ‘what would three lots of one eigth be?’

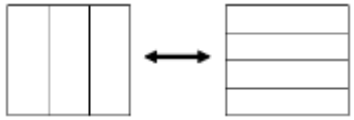
 
</td><td>When both numbers are fractions, e.g. ¼ × 1/3:
Thinking of ¼ of 1/3 gives rise to the image of quartering a picture like this:

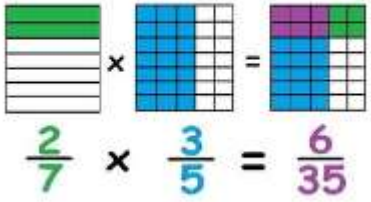
  to give this 

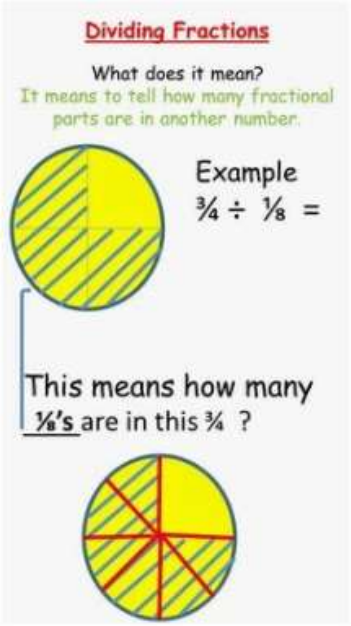
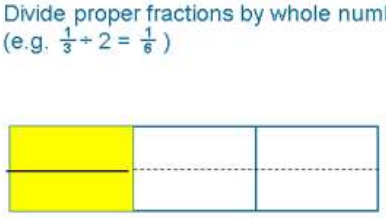
Alternatively, 1/3 of ¼ can be pictured as taking a third of a picture like this:

  to give this: 

These two images come together when you overlay two diagrams like this: (using overhead transparencies, for example).

 
</td><td>¼ × 3 is most readily appreciated as ¼ taken three times, i.e. ¼ + ¼ + ¼ or ¾ (i.e. with 3 as the multiplier) However, it could also be thought of as ¼ of 3 (i.e. ¼ as the multiplier).</td></tr>
</table>

Division of fractions **Note:** Makes it clearer to say 1/2 divided by 1/9 as 'how many ninths are in one half?' Please do NOT use 'invert and multiply' as a first strategy as chidren will not be able to reason!	Cuisenaire rods: This video (https://www.youtube.com/watch?v=4G2I8mRAuf4) goes through how to do it step by step. It's quite long and the actual division comes a few minutes in but it's worth watching from the beginning to make it clear what's going on.	 Bar models: 	

TENS FRAME IDEAS

LIFE SIZE TEN FRAME	Create a life-size ten frame in the classroom and outdoor play area. Use counters, pennies, teddies, gingerbread men, children etc.
FLASH	Flash ***ten frame*** briefly and have children write the number on a whiteboard. Using whiteboards, rather than having children say the number, ensures that all children attempt to respond and allows the teacher to assess class progress. When the response is oral, not all child responses are audible. Encourage children to share the different strategies used to find the total number of dots for cards, "How did you see it?" This can be varied by asking children to write the number and draw the pattern they saw, or by having them build the number flashed on their own blank frame.
FLASH: ONE MORE	Once children are familiar with the basic patterns, and know them automatically, flash a 10 frame or dot card and ask them to name the number that is one more than the number flashed. Variation: ask children to give the number that is two more/one less/double/ten more than the number flashed.
I WISH I HAD TEN	Flash a dot card or ten frame showing 9 or less and say, "I wish I had 10". Children respond with the part that is needed to make ten. The game can focus on a single whole, or the "wish I had" number can change each time. Variation: teacher flashes card and children write the complement of ten on individual whiteboards with dry erase markers.
I WISH I HAD 12	As above but children respond with how many more are needed to make twelve. Children should be confident in facts of 10 before this is attempted. For example to go from 8 to 12, they should realise they need 2 more to get to 10, then 2 more to 12. 2 and 2 is 4. Variation: Children draw an empty number line on their whiteboards to show the two jumps used to get to the target number.
1 MORE **1 LESS** **10 MORE** **10 LESS**	The following four prompts are written on the board: one more one less ten more ten less The teacher flashes a dot or ten frame card as the 'starting number'. The first child selects one prompt. For example, if the teacher flashes a card showing '5'
TEEN FRAME **FLASH (11-20)**	**Teen Frame Flash (11-20)** Once children are subitizing ***ten frame*** patterns 0- 10, cards showing larger numbers (i.e. more than one ten frame) should be introduced. Use mental math sessions with the following key questions: How many? How many more than 10? As children become familiar with the 'teen' patterns introduce further questions to develop number relationships. What is one more/two more than the number I flashed? What is one less/two less than the number I flashed? How far away is the number I flashed from twenty? Double the number I flash.
MULTIPLES	Flash a ***tens frame*** and ask children to give you the product if the number you flash was multiplied by 2, 5 etc.

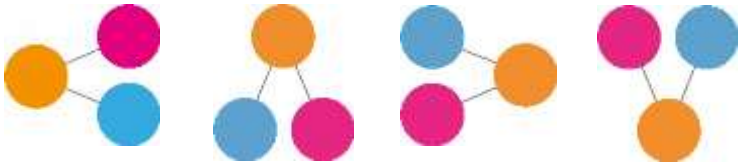
Notes for Parents

Concrete – Pictorial – Abstract

Concrete: Things you can pick up and move, for example dice, counters, shells, pebbles, straws.
Pictorial: A picture to represent mathematics, such as a calculation (printed in books or drawn).
Abstract: Numbers (1, 2, 3, 4) and symbols (+, -, x, ÷, <, >, =).

Part whole model

If you know two values, you can always find the third.



Vocabulary

Find lots of different words to say the same thing, for example add, more, increase, plus.

Problem solving

Problem solving usually involves the bar model. Encourage children to answer questions using full sentences. The bar model is usually used when solving problems.

How you can help:

Ask your child what they know about a particular number.
For example, here are some facts about 12:

- It is an even number.
- It comes after 11 and before 13.
- I can write it in numbers and words.
- I can make it using 1 ten and 2 ones.
- It is a 2-digit number.
- It is 2 more than 10 and 3 fewer than 15.
- The sum of 7 and 5 is 12.
- 5 fewer than 17 is 12.
- It has 6 factors.
- It is half of 24 (a third of 36, a quarter of 48, a tenth of 120).
- It is double 6.
- It is the product of: 3 × 4, 4 × 3, 2 × 6, 6 × 2, 1 × 12, 12 × 1.
- The sum of the digits in 12 is 3.

Encourage your child to answer word problems using full sentences, and encourage them to draw pictures and models to answer questions.

Talk about maths with your child in everyday situations, such as a shopping trip or a trip to the park.

Bar modelling

- Read the problem.
- Write a sentence for the answer, leaving a gap where the answer will go.
- Think about what is being asked and which model supports the question.
- Draw the bars.
- Partition or ‘chunk’ the bars and note which section represents the answer.
- Discuss the question and think about what is being asked.
- Write the answer in the sentence and check that the answer makes sense.
- Is there another way to draw the model and represent the question?
- What further questions could you ask using the model as a prompt?